MTH 337 Homework 4. The Butterfly Effect

15th February 2008

(a) Describe in your own words what is meant by the "butterfly effect", and why it's called that. (You may consult books and/or online resources to learn more about it.)
(b) My definition is that nearby trajectories of the dynamical system separate exponentially in time. Is this consistent with your description/definition in part (a)? Explain.
(c) I claimed that the system

$$x_{t+1} = 4x_t(1 - \frac{x_t}{3})$$

exhibits the butterfly effect, i.e. that trajectories starting close together separate exponentially - initially and on the average. To be precise, I am claiming that if the two trajectories are

$$x_0,\ x_1,\ x_2, \ldots$$

$$\tilde{x}_0,\ \tilde{x}_1,\ \tilde{x}_2, \ldots$$

with x_0 and $\tilde{x}_0$ very close to each other (but not identical), and we define their separation

$$s_t \equiv |x_t - \tilde{x}_t|, \qquad t = 0,\ 1,\ 2, \ldots,$$

then, until the separation is so large it can't get any larger, I'm saying

$$s_t \approx s_0 a^t$$

for some constant, a, which is greater than 1. Alternatively, since any positive number, a, can be written as e^α for some α (in fact $\alpha = \log a$), we can also write the above equation as

$$s_t \approx s_0 e^{\alpha t}.$$

for some constant, α, which is positive.
Validate or invalidate my claim as follows.

Generate a pair of trajectories of this system, starting anywhere between 0 and 3 (excluding 0 and 3 themselves), with the initial separation of the two trajectories being 10^{-13}. (Be sure to set Maple's precision to at least 16 digits: `Digits:=16` or greater.)

(i) Make a plot that shows x_t versus t for both trajectories on the same plot. Use red for one and blue for the other. Plot lines (not points) so it's easier to follow the trajectories.
(ii) Make another plot (the one you did in the Lab on 2/14/08) that shows the log of the separation of the two trajectories versus t.
Choose your range of t large enough to see the separation plateauing out, but small enough that you can see clearly what is happening while the separation is growing.
Discuss why your plots support or contradict my claim.
(c) If you find my claim supported by your data, attempt to find the "best fit" value of a or of α.
(d) For extra credit, if you have time, explore, describe what happens in the system

$$x_{t+1} = rx_t(1 - \frac{x_t}{3})$$

for some values of r other than 4.