MTH 444/544 FOAM II EXAM #2

Tuesday, March 28, 2006. *For maximum credit, show all work, mark clearly which part of which problem you are answering, write legibly, and try to make it very clear what you are doing. Instant grading/correcting is available.*

Question	Max points	Score
1	4	
2a	2	
2b	1	
2c	2	
2d	5	
3a	2	
3b	4	
4a	2	
4b	3	
4c	3	
4d	2	
4e	3	
4f	2	
4g	2	
5a	3	
5b	5	
TOTAL		

1. (4pts) Fill in the blanks in the table below. In handwritten vector notation use an underline to denote vectors and a double-underline to denote 2-index tensors respectively.

indicial notation	vector notation
u_j	
p	
	$p\mathbf{I}$
	$\mathbf{T}$
$p_{,i}$	
	$\nabla \cdot \mathbf{u}$
	$\nabla \cdot \mathbf{T}$
$\phi_{,ii}$	
$\phi_{,kk}$	
	$\nabla^2\mathbf{u}$
$u_{r,ri}$	
	$\mathbf{c}=\mathbf{a} \times \mathbf{b}$

2. (a) (2pts) Write down the equation of momentum balance for an isotropic compressible viscous fluid (assuming viscosity parameters are constant).

(b) (1pt) How does the equation in (a) simplify if the the fluid is incompressible?

(c) (2pts) Write down the stress tensor in an isotropic viscous fluid in terms of the velocity gradient.

(d) (5pts) Derive the equation you wrote in part (a) by plugging the formula you wrote in part (c) into the following:

$$\rho \frac{D\mathbf{u}}{Dt} = \rho\mathbf{b} + \nabla \cdot \mathbf{T} \ .$$

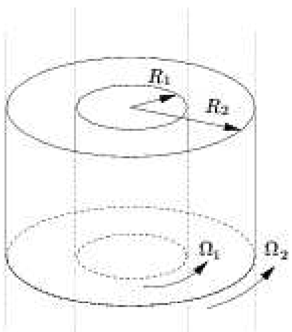
Begin by converting the equations above to indicial notation. You may use the fact that $\mathbf{T}$ is symmetric.

3. (a) (2pts) Write down the velocity of the fluid in Rayleigh impulsive flow in terms of ν, U, y, and t.

(b) (4pts) Find the tangential component of the stress (force per unit area) on the plate.

Please apply the chain rule correctly, and recall how to differentiate an integral with respect to its upper endpoint: $\frac{d}{dz}\int_0^z f(x)dx = f(z)$.

4. Consider the flow of a viscous incompressible fluid between two infinitely long coaxial cylinders of radii R_1 and R_2 rotating at angular speeds Ω_1 and Ω_2 repsectively. Find the velocity field of the form $\mathbf{u}(r,\theta,z) = u(r)\hat{\theta}$ that satisfies momentum balance (Couette flow).



Follow these steps:

(a) (2pts) Write down ∇ and ∇^2 in cylindrical coordinates.

(b) (3pts) Express $(\mathbf{u}\cdot\nabla)\mathbf{u}$ as simply as possible in the case $\mathbf{u}(r,\theta,z) = u(r)\hat{\theta}$. (Don't forget that $\frac{\partial\hat{\theta}}{\partial\theta} = -\hat{\mathbf{r}}$, $\frac{\partial\hat{\mathbf{r}}}{\partial\theta} = \hat{\theta}$, while $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial z}$ of $\hat{\mathbf{r}}$, $\hat{\theta}$ and $\hat{\mathbf{z}}$ are all $\mathbf{0}$, as is $\frac{\partial\hat{\mathbf{z}}}{\partial\theta}$.)

(c) (3pts) Express $\nabla^2\mathbf{u}$ as simply as possible in the case $\mathbf{u}(r,\theta,z) = u(r)\hat{\theta}$.

(d) (2pts) Show that the $\hat{\mathbf{r}}$ component of momentum balance determines the pressure, and that the $\hat{\theta}$ component requires that the function $u(r)$ satisfy the ODE $r^2u'' + ru' - u = 0$.

(e) (3pts) Find the general solution of the above. (Try $u(r) = r^\alpha$.)

(f) (2pts) State the appropriate boundary conditions on $u(r)$.

(g) (2pts only: leave this till the end!) Finish the determination of $u(r)$ (in terms of R_1, R_2, Ω_1, Ω_2).

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5. (a) (3pts) Write down any formula(s) you know about Poiseuille (pipe) flow. Also write down the values of any physical parameters of *air* that you know.

(b) (5pts) You can easily verify that you can breath with reasonable comfort through a drinking straw, radius R =3mm, length 20cm. But if the straw were longer, you might find it hard to draw enough air through it. Earlier in the semester, we estimated experimentally that the maximum breathing-in pressure (suction) for a human (me) is about ΔP=4000 Pa. Assuming that I must breath in and out at least 0.5 liter (1 liter= $(0.1\text{m})^3$) of air every 5 seconds to survive, estimate the maximum length, L, of a drinking straw I could breath through, assuming Poiseuille flow in the straw. If you cannot solve this completely, outline how it can be done using the formulas and data you (hopefully) gave in (a).