455 Day 2 Maple Exercise

January 20, 2005

Today we will use Maple to explore the dynamics of the "logistic map"

$$x_{t+1} = r_0 x_t (1 - x_t)$$

as the parameter r_0 is varied. Recall that this map arises as a simple model for a population with non-overlapping generations, when the per-capita growth rate r is taken to decrease linearly with the size of the population, x: $r = r_0(1 - x)$.

Throughout this exercise, you can find out how to do things in Maple by consulting the Maple guides on the class website: *http://orange.math.buffalo.edu/455/*. You may find it useful to read through those guides quickly now.

Save your work frequently (in case Maple crashes), and use the "Classic Worksheet (mws) format for compatibility with older versions of Maple that are less of a memory-hog.

1 Plotting iterates of the logistic map

(Use of `seq` and `plot`, and defining Maple functions.) The `seq` command is one of the most useful in Maple. It is used to create sequences (comma-separated lists) of any kinds of objects (numbers, mathematical expressions, plots, etc.). For example, try generating a geometric sequence like this:

```
seq(r^t,t=0..10);
```

and then the following (Use Shift-Enter to create multi-line execution groups)

```
r:=2.0;
seq(r^t,t=0..10);
```

You can plot a set of "[x,y]" data points like this:

```
r:=2.1;
my_data_set:=[seq([t,r^t],t=0..30)];
plot( my_data_set, style=point, labels=["t","x"]);
```

Now let's do the same thing except iterating the logistic map instead of the geometric map. This requires us to learn about defining functions, and using the "for" loop.

```
unassign('r'): %This clears the assignment of the value 2.1 to the symbol r
```

```
f:=(r,x)->r*x*(1-x);
```

Once the function f is thus defined, we can use it in this way:

```
f(2.3,0.1);
```

giving the value $2.3 \cdot 0.1 \cdot (1 - 0.1) = 0.23 \cdot 0.9 = 0.207$. Now we can generate a sequence of iterates of the map as follows:

```
x:=0.1;

pt[0]:=[0,x];

for t from 1 to 20 do
  x:=f(2.3,x); % Here we overwrite x with f(x)
  pt[t]:=[t,x];
end do:  unassign('t'):

my_sequence:=[ seq( pt[t], t=0..20 ) ];

plot(my_sequence, style=point, labels=["t","x"]);
```

Note how the population slightly overshoots the steady-state, or fixed point, at about $x = 0.56$ before settling to fixed point. In the above, we plotted 20 iterates for $r = 2.3$. Now try running the above cell again after changing the value of r and/or the number of iterates. Describe some of your observations on the pink sheet.

2 Making cobweb diagrams

In Section 1, you have been plotting $x_t$versus t. As I described in class, in order to understand the dynamics it is helpful instead to plot x_{t+1}versus x_t, for $t = 0, 1, 2, ...$, drawing verticals to the graph of f and horizontals to the graph of the identity, id.

Maple ingredients you will need, in addition to what you learned above, are:

(i) Plotting the graph of a function:

```
plot(f(2.3,x),x=0..1,color=red);
```

(ii) Using display to combine several plots (see Maple for Graphics).

```
pf:=plot(f(2.3,x),x=0..1,color=red): %Note the use of the colon to suppress printing of plot description

pid:=plot(x,x=0..1,color=blue):
```

```
plots[display](pf,pid);
```

(iii) Creating a Maple `procedure` (like a Maple function, but when evaluation needs several steps):

```
cobweb := proc( f, r, x0, n )

% steps of procedure go here

return [the final displayed plot];

end proc:
```

The "arguments" of this procedure are the name of the map you want to iterate, the value of its parameter, the starting value of x and the number of iterates you want. This is pretty challenging. See what you can do!